

Superrelativity as a unification of quantum theory and relativity

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Abstract

Principle of “Superrelativity” has been proposed in order to avoid the contradiction between principle of relativity and foundations of quantum theory.

Solutions of a newly derived non-linear Klein-Gordon equation presumably may be treated as primordial nonlocal elements of quantum theory.

It is shown that in the framework of $CP(N - 1)$ model supplementary elements which are non-local in spacetime but local in the projective Hilbert space permit us to avoid at least one of the main difficulties of quantum theory—the necessity to relate the “reality” of a quantum state with a measuring process. In the framework of superrelativity the geometry of the projective Hilbert space (Fubini-Study metric and connection) together with the non-linear wave equation are full and close quantum scheme.

Key words: wave packet, superposition principle, projective Hilbert space, scalar field, equivalence principle, curvature of the space of pure quantum states, tangent fiber bundle, gauge field

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1 Introduction

All attempts to achieve a reasonable generalization of the quantum theory which are based on the Einstein’s principle of general relativity (GR) lead to major difficulties. The reason is that in the framework of GR only the spacetime structure has been modified, but not the quantum state space. That is one treats quantum particles as a material point in a spacetime and internal degrees of freedom are expressed by the evolution of second quantized amplitudes of a probability in the state space. This point of view came from Dirac’s classical articles [1,2]. In his work Ref. [2],

the introduction of internal degrees of freedom for an electron (spin) is based on the principle of special relativity. Dirac treats the “doubling” of the number of states as an evidence of the non-locality of the electron. However, he formulates the local problem to find a Hamiltonian linear in the operator of momentum \hat{p}_μ for the point charged electron. But one suspects that besides the spin degree of freedom, there are some “hidden degrees of freedom” which describe a spatial distribution of both charge and mass. Therefore, I think we should *modify* two of Schrödinger’s original ideas:

1. Wave description of elementary particles as wave packets [3],
2. Wave propagation in the configuration space [4,5].

We propose here a simple possibility for a “shaping” of a stable wave packet (“droplet”) from solutions of the linear Klein-Gordon equation by the action of geodesic flow in the Hilbert projective space $CP(N - 1)$ [6-9].

In principle, it is impossible to distinguish “external” and “internal” degrees of freedom of a quantum system. *Therefore, to my mind, one must take into account an entanglement of the spacetime and the state space of a quantum particle.* Supergravity realizes it in so-called superspace. Instead, I propose to describe quantum particles in a single **projective Hilbert space** $CP(N - 1)$. *It is a quite different approach to the non-distinguishability of quantum systems and the decoherence of isolated “bits”. In this approach spacetime arises as an auxiliary entity for a description of motion of quantum integrals.* Moreover, in an attempt to unify of fundamental interactions we try to achieve this just in the $CP(N - 1)$. That is, a distance in this space is **the square root of an action** and not spacetime interval [7-9]. One should find a new geometric structure with this *interval of action* for an imbedding of both internal and external kinds of symmetry.

$SU(N)$ symmetry is an example of so-called internal symmetry of elementary particles. This symmetry is broken up to the isotropy group $H = U(1) \otimes U(N - 1)$ of the pure quantum state. We will show that a character of this break-down is connected to the geometric structure mentioned above as the coset $SU(N)/S[U(1) \otimes U(N - 1)]$ is connected with the projective Hilbert space $CP(N-1)$. This geometry is associated with the spacetime distribution of physical carriers of charges, masses etc., i.e. elementary particles.

2 What is our goal?

Goal We wish to create a concentrated self-interacting field configuration “droplet” which has its own surrounding gauge field (intrinsic potential). It should play the role of the model of non-local quantum particles in the framework of the causal approach to quantum theory [6-9,11].

Now, at the 70th birthday of Quantum Mechanics, it is clear that the price of success must be very high. Nevertheless we ought to pay, if we want to reach a

reasonable comprehension of the quantum entity.

Price We should forget about the spacetime priority. Quantum state space and states themselves are fundamental elements of the “quantum essentiality”. That is our physical *dynamical* spacetime arises as a geometry of moving nonlocal but concentrated quantum particles. Therefore we have to find some geometry reflecting both quantum features and the possibility of the quasiclassical approximation in a natural way.

As a matter of fact this point of view is neither shocking nor novel. For example, Y. Ne’eman wrote that the “quantum reality” of complex quantum amplitudes is “represented by Hilbert space, rather than by spacetime” [10]. But ordinary Hilbert space is closely related to ordinary spacetime and it must be clear how to unify them. One of the ways is well known now. This is supersymmetry and its local version - supergravity. But this method of unification acts as if both spacetime and the space of internal degrees of freedom themselves were independent entities [11].

Method I propose a new geometric framework in order to unify both “external” and “internal” degrees of freedom which I will call “Superrelativity” (SuperR). *The SuperR physically means that there is a unified self-interacting physical field of de Broglie-Schrödinger-Bohm which is associated with the geometry of the projective Hilbert space $CP(N-1)$. This space has a constant positive holomorphic sectional curvature. The curvature gives an intrinsic interaction of modes of extended quantum particles. Since the projective Hilbert space $CP(N-1)$ is a homogeneous manifold, there are both local and global conservation laws. Hereof there arise so-called local (in $CP(N-1)$) dynamical variables which depend on the state of a quantum system (local coordinates); dynamical variables in special or in general relativity depend on the state of a motion of the material point as well.*

3 Principle of “Superrelativity”

I try to achieve of the ‘peaceful coexistence’ between principles of Einstein’s relativity (locality and determinism) and quantum theory which is incompatible with local supplementary parameters. Note, this should be done in the framework of a *non-linear approach*. This requires a deep reconstruction of the relationships between spacetime structure and the quantum state space (in my model I used the complex projective Hilbert space of indefinite dimensionality $CP(\infty)$ or, in some approximation, the complex projective Hilbert space $CP(N-1)$ of finite dimension). Briefly speaking, I tried to expand a wave description on ‘quantum particles’ (in the spirit of de Broglie-Schrödinger-Bohm [12]) in the framework of a more wide and “soft” structure than spacetime, even than the curved Einstein spacetime of general relativity. The physical meaning of this structure (from the mathematical point of view it is the tangent fiber bundle over $CP(\infty)$ or $CP(N-1)$) may be expressed in a principle of “Superrelativity”.

In order to realize this principle we should introduce two fundamental notions. They are “state” and “transition” of a quantum system. These notions should be used instead of “material point” and “event” in special relativity (SR) or in GR. Note that the spacetime structure “grows” from the geometric structure of the state space. That is at the microlevel spacetime coordinates (in Einstein’s sense) have a merely interpolating sense (in the reference Minkowski spacetime) at best and do not exist at worst.

Superrelativity is based on simple physical facts:

1. *Every event in the sense of special or general relativity is a quantum transition.* Therefore, one cannot invoke *only* our spacetime experience and should deal with a state space. For instance, in an arbitrary chosen point B of the Minkowski spacetime there does not exist a natural notion of the “same direction” relative to the initial direction in the initial point A . At every point one can define the “direction” as the direction of some physical vector field. Therefore, a comparison of directions must be reduced to the comparison of the vector fields. As a matter of fact one should establish a law of “parallel transport” of quantum dynamical variables of the vector fields configuration. This law takes place in the state space of the field configuration and not in spacetime itself. It is a generalization of the well known Pancharatnam problem of the comparison of polarizations of two beams [11].

2. *The structure of the state space reflects symmetries of real quantum particles.* $SU(N)$ symmetry is an example of so-called internal symmetry of elementary particles. Here we seek only the simplest possibility, assuming that this symmetry is broken up to the isotropy group $H = U(1) \otimes U(N - 1)$ of the pure quantum state.

We assume that the content of the “superrelativity principle” is as follow: *The general unitary motion of pure quantum states may be locally reduced to geodesic motion in the projective Hilbert space by introduction of some gauge (compensation) field for self- preservation of the geodesic field configuration (droplet). Herein a surrounding field arises; we try to identify it with a “physical field” in the ordinary spacetime.* Note, that “superrelativity” assumes some “superequivalence” of unified physical field and geometric properties of the base manifold (in our model it is projective Hilbert space which is equipped with the generalized Fubini-Study metric [6-9,11]). It is useful to compare the equivalence principle of Einstein and “superequivalence” principle.

The Einstein’s equivalence principle has often been subjected to criticism. It is correct that it is fulfilled only locally. It is correct that the absence of a gravitation field at a point implies a zero value of the Riemann tensor of the spacetime and this condition does not depend on the character of an observer’s motion. There exists an opinion, however, that this principle should be discarded. If we understand this principle *literally* as the equivalence of curvature of the spacetime (gravity) and the arbitrary “physical fields” as a reason for the accelerated motion, then perhaps this really should be done. But I think that we ought to take into account the *general aspiration* of Einstein. I mean that he tried to build a *unified field theory*. Everyone

knows it must be a quantum theory. In the framework of this quantum theory all fields have a unified nature and therefore *a novel equivalence principle in Einstein's spirit* should concern *quantum states*, not material points. Such notions as “accelerated motion” and “uniform motion” are no longer applicable to quantum states. Hence, one must invent *some new classification of motion of quantum states* (i.e. classification of quantum transitions). Then we should formulate a “superequivalence” principle on the base of this classification. In our case this classification will be based upon a geometrically invariant distinction of local and global conservation laws in the projective Hilbert state space. In accordance with this intrinsic classification, we have two kinds of motions:

A. Unitary “rotations” of the “ellipsoid of polarization” under transformations from the isotropy group $U(1) \otimes U(N-1)$ of the pure quantum state (Higgs modes).

B. Geodesic “motion” of pure quantum states in the projective Hilbert space as a hidden (virtual) transition under transformations from the coset $SU(N)/S[U(1) \otimes U(N-1)]$. They are pure “deformations” of the “ellipsoid of polarization” (Goldstone modes).

Pure quantum states of “isolated” quantum systems are rays $\{|\Psi\rangle\} = A \exp i\alpha \sum_{a=0}^{N-1} \Psi^a |a, x\rangle$ and they belong to the projective Hilbert space $CP(N-1)$ [13]. There are appropriate local coordinates $\pi_{(b)}^i$ of the chart atlas $U_b = \{|\Psi\rangle = \sum_{a=0}^{N-1} \Psi^a |a, x\rangle : \Psi^b \neq 0\}$ in $CP(N-1)$. For $b=0$ one has

$$\pi_{(0)}^i = \Psi^i / \Psi^0, \quad (3.1)$$

where $1 \leq i \leq N-1$. Then the fundamental tensor of Fubini-Study metric in $CP(N)$ is

$$g_{ik*} = 2\hbar \frac{(1 + \sum_{s=1}^{N-1} |\pi^s|^2) \delta_{ik} - \pi^{i*} \pi^k}{(1 + \sum_{s=1}^{N-1} |\pi^s|^2)^2}. \quad (3.2)$$

[13-15]. At first sight the superposition principle permits one to work with the relative amplitudes and relative phases, i.e., one must forget about modulus of the wave function. However perhaps the superposition principle serves merely as a very good approximation. This is a quite natural assumption if we try to build a *nonlinear quantum theory where this principle, of course, does not act*. Then the modulus has a physical meaning and, hence, should be taken into account. I shall assume that projective symmetry is broken up to the symmetry of the Kähler manifold with metric (3.3). It may be done by using the generalized Fubini-Study metric tensor G_{ik*} in $CP(N-1)$ [6-9,11], which is defined by the formula

$$G_{ik*} = 2\hbar R^2 \frac{(R^2 + \sum_{s=1}^{N-1} |\pi^s|^2) \delta_{ik} - \pi^{i*} \pi^k}{(R^2 + \sum_{s=1}^{N-1} |\pi^s|^2)^2}. \quad (3.3)$$

The real part of (3.3) is a Riemannian structure and the imaginary part is a symplectic one. In addition, the natural connection (see below) determines an intrinsic gauge

potential. The symplectic structure plays an important role in the geometric phase. The Riemannian structure and curvature of $CP(N-1)$ is closely connected with the density of Schrödinger's wave, because the *generalized Fubini-Study metric* (3.3) can then be regarded as an induced Riemann metric of $CP(N-1)$. It is obtained by the “stereographic projection” of rays of the Hilbert space $C(N)$ from the “density sphere” $\sum_{a=0}^{N-1} |\Psi^a|^2 = R^2$ with radius R . Therefore the well known geometric interpretation of Planck's constant is appropriate as a normalizing factor for the radius of the sphere S^{N^2-2} in the $AlgSU(N)$ [15] for average $\langle A \rangle = \frac{\langle \Psi | \hat{A} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$. But internal (Riemann) geometry takes place on the “density sphere” S^{2N-1} in the Hilbert space. That is we have a *spectral parameter R for the “foliation” of the tangent bundle over $CP(N-1)$ where the unified fundamental interaction acts*. As a matter of fact it is the lift of a “trace of the quantum transition” in the base $CP(N-1)$ into the tangent fiber bundle of the “experimental environment of external fields”. Then the semiclassical limit may be achieved if $R \rightarrow \infty$ [8]. We will show (see below) that this limit physically may be achieved very easily for a *finite* value of R . Hence, I think it more natural to identify the curvature of state space not with Planck's constant but with fine structure constant $c = 1/R^2 = \alpha$. In our case Planck's constant is merely a normalizing factor as well as in Ref. [15]. We, therefore, do not treat the “semiclassical limit” in terms of the limit as Planck's constant \hbar tends to zero, but as R tends to infinity. That is we can avoid the conclusion that “radius of holomorphic sectional curvature goes to zero as $\hbar \rightarrow 0$ ” which is very paradoxical [15]. We have here an example of a different kind of non-linearity than in Weinberg approach [16,17].

4 Generalized Pancharatnam connection

The generalized Pancharatnam's problem of comparison of the phases of beams is akin to the Shapere-Wilczek approach to the comparison of shapes of deformable bodies in their gauge kinematics [18]. In our case the projective Hilbert space $CP(N-1)$ takes the place of the space of some “unlocated shapes” [11]. Of course, it is impossible to understand the “shape of wave packet” - droplet,- literally as the shape of the quantum particle in the real spacetime. The droplet is a geodesic (periodic) deformation of Fourier components of the initial solution of Klein-Gordon equation under transformations from the coset $SU(N)/S[U(1) \otimes U(N-1)]$. That is the problem which arises in our case and should be stated as follows: *what are the dynamical variables that correspond to sequence of deformations of solution of the initial linear equation?* Here $CP(N-1)$ is a base manifold and $U(1) \otimes U(N-1)$ is the structure group in a fiber.

The problem of the comparison of real spatial shapes, which undergo large deformations, has not yet been solved [18]. But in our case the “instantaneous shape of a wave packet” is represented by *known vector fields of polarizations as functions of relative Fourier components themselves* and deformations of this “shape of ellipsoid

of polarization” lay in the coset $SU(N)/S[U(1) \otimes U(N-1)]$ [6-9]. That is the natural connection in $CP(N-1)$

$$\Gamma_{kl}^i = -2 \frac{\delta_k^i \pi^{l*} + \delta_l^i \pi^{k*}}{R^2 + \sum_s^{N-1} |\pi^s|^2} \quad (4.1)$$

which corresponds to the Fubini-Study metric (3.3), can help us to compare these shapes. Perhaps the most obvious example of the same kind is a representation of the polarization states of photons or electrons by points of the Poincaré sphere [19]. The “shape” in this case is indeed the shape of the ellipse of polarization. The ellipse conserves its own shape along every “parallel of latitude” but the orientation of this ellipse is smoothly and periodically changing. We will show that *the connection (4.1) determines a quite natural intrinsic gauge potential of a local frame rotation in a tangent space of $CP(N-1)$ and, therefore, renormalization of dynamical variables. Relationships between Goldstone’s and Higgs’s modes arise in an absolutely natural way also. Namely, the shape of the graphic (4.1) is similar to the well known artificial potential surface $V = \lambda^2 |\pi|^4 - \mu^2 |\pi|^2$ for the illustration of the spontaneously broken symmetry, however our “potential” (4.1) is finite anywhere.*

5 Carrier of Dynamical Variables

The general form of Newton’s second law is indifferent to the type of force. Only development of electromagnetic theory, that is a particular kind of force, leads to the relativistic generalization of the classical mechanics. The Schrödinger equation of ordinary quantum mechanics is indifferent to the choice of a potential also. However, at short distances this potential can not be arbitrary. Therefore, it is not enough to use the Hertz metric of configuration space, which contains an arbitrary potential of the “environment” [4,5]. Underlying “hidden” degrees of freedom, connected with internal symmetries of elementary particles, should be used for “shaping” an intrinsic potential which is a “particle” itself. This approach coincides in general with de Broglie’s idea [20]. I will build our model in the spirit of two main approaches:

1. Schrödinger’s method of coherent states for stable wave packet “shaping” [3],
2. Bohm approach to the nonlinear origin of fundamental equations of elementary particles [12].

The essential new element of our approach is the action of geodesic flow in the configuration space on the relative Fourier components $CP(N-1)$. Since the principle of least action arises in $CP(N-1)$ as a principle of least curvature, geodesic flow in $CP(N-1)$ plays an important role [8,9,11]. Its integral curves (geodesics) are stable and closed (periodic).

It should be clear that we are not ready yet to present a quite consistent theory. Our aim is to show how we can move toward this desirable goal. Then we can introduce the notion of a “reference Minkowski spacetime” as an analogy of a “screen

2-space” on our PC. The mouse has really 7 degrees of freedom in the original 3-space, but the pointer has only 2 degrees of freedom. I think in the quantum “reality” we have the same situation. Namely, the quantum field system (mouse) can have in general an indefinite number of degrees of freedom, but the “centrum of mass” of the droplet, which takes the place of the “pointer of quantum transition under registration”, has only 4-spacetime degrees of freedom. The connection between the “quantum mouse” and “pointer” may be realized by some field model in spirit of Bohm [12].

As an example consider the effective self-interaction scalar field Φ in “reference Minkowski spacetime” [11]. That is we neglect any dynamical effects (like effects of the spacetime curvature in general relativity) in *this manifold*. The physical spacetime should arise as a geometry of moving droplet where these dynamical effects may be observed. We wish to write a Lagrangian for non-linear interaction, which is Poincaré invariant, and, therefore, choose a field which depends only on the “radial” variable ρ , i.e. $\Phi = \Phi(\rho)$, where $\rho^2 = x_\mu x^\mu$ and x^μ corresponds to a relative spacetime coordinate (emerging, for example, from some underlying dynamical model for self-interaction). Since $\Phi_{,\mu} = \frac{\partial \Phi}{\partial x^\mu} = \frac{x_\mu}{\rho} \frac{d\Phi}{d\rho}$ and $\Phi^{,\mu} = \frac{\partial \Phi}{\partial x_\mu} = \frac{x^\mu}{\rho} \frac{d\Phi}{d\rho}$, a Lagrangian density may be written as

$$\mathcal{L} = \frac{1}{2} \Phi_\mu^* \Phi^\mu - U(\Phi(\rho)) = \frac{1}{2} \left| \frac{d\Phi}{d\rho} \right|^2 - U(\Phi(\rho)), \quad (5.1)$$

where we have assumed a general form for the effective self-interaction term $U(\Phi(\rho))$. The equation of motion of the scalar field acquires the form of the ordinary differential (nonlinear in general) equation

$$\frac{d^2 \Phi^*}{d\rho^2} + (3/\rho) \frac{d\Phi^*}{d\rho} + 2 \frac{\partial U(\Phi(\rho))}{\partial \Phi} = 0 \quad c.c. \quad (5.2)$$

If the potential $U(\Phi(\rho))$ has the form $U(\Phi(\rho)) = \frac{1}{2}(mc/\hbar)^2 \Phi^* \Phi = \frac{1}{2} \alpha^2 |\Phi|^2$, then (5.2) is the linear Lommel equation

$$\frac{d^2 \Phi^*}{d\rho^2} + (3/\rho) \frac{d\Phi^*}{d\rho} + \alpha^2 \Phi^* = 0, \quad c.c. \quad (5.3)$$

for which a solution is the Bessel function [21] $\Phi = \rho^{-1} J_{-1}(\alpha\rho)$. We should note that in the timelike sector of the spacetime where $\rho'^2 = -\rho^2 > 0$ corresponds to the de Broglie equations which has a solution which is the modified Bessel functions $\chi = \rho'^{-1} I_{-1}(\alpha\rho')$. However a “deformation” of these solutions into solutions of some *effectively nonlinear Klein-Gordon or de Broglie equation* by the geodesic flow is interesting for our purpose. This point is a crucial difference between our approach and, say, a model of Rubakov-Saha [22]. I wish to emphasize the connection of our model with the so-called “off-shell models” [23].

If we choose the the classical radius of the electron $r_0 = \frac{e^2}{mc^2}$ as the unit of distance scale $\rho = x r_0$, then $(mc/\hbar)^2$ in (5.3) becomes the fine structure constant $\alpha = \frac{e^2}{\hbar c}$. Let's suppose $y = (\rho/r_0)^2$.

It is well known that on the interval $(-\infty, \infty)$ the set of orthogonal Hermitian functions $|n, y\rangle = \phi_n(y) = (2^n n! \sqrt{\pi})^{-1/2} \exp(-y^2/2) H_n(y)$ is complete and one can represent a solution of (5.3) in the y -variable as a Fourier series

$$|\Phi\rangle = x^{-1} J_{-1}(\alpha x) = \sum_{k=0}^{\infty} \Phi^k \phi_k(x^2) = \sum_{k=0}^{\infty} \Phi^k |k, y\rangle, \quad (5.4)$$

where

$$\Phi^m = \frac{-1}{2^m m! \sqrt{\pi}} \int_{-\infty}^{\infty} dy \exp(-\frac{y^2}{2}) H_m(y) y^{-1/2} J_1(\alpha y^{1/2}) \quad (5.5)$$

Our geodesic flow acts on these Fourier components. It is convenient to transform the state covector (5.5) to the “vacuum” form $\Phi_0 = \exp(i\omega) ||\Phi|| (1, 0, \dots, 0, \dots)$ by a set of matrices \hat{G} , obtained as follows [6-9]. It is easily seen that for a vector of this form, geodesic flow is generated by a general linear combination of “creation-annihilation” operators

$$\hat{B} = \begin{pmatrix} 0 & f^{1*} & f^{2*} & \cdot & \cdot & \cdot & f^{N-1*} \\ f^1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ f^2 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f^{N-1} & 0 & 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix}. \quad (5.6)$$

The flow is then given by the unitary matrix $\hat{T}(\tau, g) = \exp(i\tau \hat{B}) =$

$$\begin{pmatrix} \cos \Theta & \frac{-f^{1*}}{g} \sin \Theta & \cdot & \cdot & \cdot & \frac{-f^{N-1*}}{g} \sin \Theta \\ \frac{f^1}{g} \sin \Theta & 1 + [\frac{|f^1|}{g}]^2 (\cos \Theta - 1) & \cdot & \cdot & \cdot & \frac{f^1 f^{N-1*}}{g^2} (\cos \Theta - 1) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{f^{N-1}}{g} \sin \Theta & \frac{f^{1*} f^{N-1}}{g^2} (\cos \Theta - 1) & \cdot & \cdot & \cdot & 1 + [\frac{|f^{N-1}|}{g}]^2 (\cos \Theta - 1) \end{pmatrix}, \quad (5.7)$$

where $g = \sqrt{\sum_{k=1}^{N-1} |f^k|^2}$, $\Theta = g\tau$ [6,9,11]. The form of the periodic geodesic “deformation” of the initial solution of Eq.(5.3) is represented by the formula

$$|\Psi(\tau, g, x)\rangle = \sum_{m,n=0}^{\infty} |n, x\rangle \Phi^m [\hat{G} \hat{T}(\tau, g) \hat{G}^{-1}]_m^n. \quad (5.8)$$

That is, we have geodesic “generation” of nonlocal droplet with finite action. We try to build a quantum dynamical variables over Fourier modes of some field like in Ref. [24] in distinction from Schrödinger’s wave function of coordinates of material points

[4,5]. But instead of a cavity massless scalar field of Ref. [24] we used the massive “Lorentz-radial” scalar field $\Phi(x)$.

The uniform rotation (5.7) of a state vector in the Hilbert space should be connected with a motion of the local coordinate (3.1) (Goldstone modes). On the other hand every geodesic could be rigidly transformed from one to another by transformations from the isotropy group $U(1) \otimes U(N-1)$ of the “vacuum” state, because $CP(N-1)$ is a “totally geodesic manifold” [14] (Higgs modes). That is one can identify any geodesic as a “rigid framework” for the shape of the stable wave packet - droplet. They look like “closed strings” in $CP(N-1)$. In particular cases it may be transformed into a geodesic in $CP(1)$. Then $\Pi = R \exp(i\alpha) \tan l$ is a solution of the equation of a geodesic in $CP(1)$

$$\frac{d^2\Pi}{dl^2} - \frac{2\Pi^*}{R^2 + |\Pi|^2} \left(\frac{d\Pi}{dl}\right)^2 = 0. \quad (5.9)$$

In the general case $CP(N-1)$ we have $\pi^i(\lambda) = R(f^i/g) \tan \Theta$ for the *uniform rotation* of the “vacuum” state Φ_0 in the original Hilbert space C^N with the rate g . However, *relative to the natural measure in $CP(N-1)$, i.e. relative to the length $l(\pi, \pi^*)$ of a curve (action), this “rotation” is far from uniform.* The relationship between these rates may be expressed by following equation

$$\frac{d^2\Theta}{dl^2} + 2[1 + 2/R](\frac{d\Theta}{dl})^2 \tan \Theta = 0. \quad (5.10)$$

We can present a numerical solution of this equation as a dependence of the uniform rotation parameter τ on the natural (canonical) parameter l . It may be shown that $\lim_{l,R \rightarrow \infty} \Theta(l, R) = \pi/2$. *That is orthogonality of pure states for large action may be achieved just in the “semiclassical limit” [8]. Therefore the geometric structure (curvature) of the Hilbert projective space has an essential physical meaning in terms of decoherence.*

In order to define the surrounding field of our scalar carrier-droplet we should introduce new important notions.

6 Local Dynamical Variables and Field Equation

The problem of building of consistent quantum dynamical variables (time and frequency) using the underlying symmetries of quantum fields was raised by M.-T.Jaekel and S.Reynaund [25]. The main aim of this work is to clarify the notion of some kinds of spacetime transformation in a framework of ‘a novel conception of spacetime which would be free from its difficulties inherited from classical physics’. It is an absolutely legitimate question but it seems to me to require a generalization. In comparison with Ref.[25] there are two differences in our approach to a similar problem: the first one is

our conception of the *quantum transition* instead of “event” of SR or GR and $CP(N-1)$ construction as fundamental physical structure; the second one is a 4-dimensional spacetime structure instead of 2-dimensional model.

Definition 1 *Local (state-dependent) dynamical variables are tangent vector fields to the $CP(N-1)$ associated with one-parameter subgroups of unitary transformations [6-9].*

That is, we now refer to the term “local” as a fact of a dependence on the coordinates (3.1) in the $CP(N-1)$ as in Ref. [26]. We should find the relationship between the linear representations of $SU(N)$ group by an “polarization operator” $\hat{P} \in AlgSU(N)$ which does not depend on the state of the quantum system and the nonlinear representation (realization) of the group symmetry in which the infinitesimal operator of the transformation depends on the state. In the linear representation of the action of $SU(N)$ we have

$$|\Psi(\epsilon)\rangle = \exp(-i\epsilon\hat{P})|\Psi\rangle. \quad (6.1)$$

For a full description of a group dynamics by pure quantum states, we shall use coherent states in $CP(N-1)$. Let's assume \hat{P}_σ is one of the $1 \leq \sigma \leq N^2-1$ directions in the group manifold. Then

$$D_\sigma(\hat{P}) = \Phi_\sigma^i(\pi, P) \frac{\delta}{\delta\pi^i} + \Phi_\sigma^{i*}(\pi, P) \frac{\delta}{\delta\pi^{i*}}, \quad (6.2)$$

where

$$\Phi_\sigma^i(\pi; P_\sigma) = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon P_\sigma)]_m^i \Psi^m}{[\exp(i\epsilon P_\sigma)]_m^k \Psi^m} - \frac{\Psi^i}{\Psi^k} \right\} = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \{ \pi^i(\epsilon P_\sigma) - \pi^i \} \quad (6.3)$$

are the local (in $CP(N-1)$) state-dependent components of generators of the $SU(N)$ group, which are studied in [6-9]. So in the general case for group transformations of more than one parameter we have a vector field for the group action on $CP(N-1)$ by some set of dynamical variables $\hat{P}_1, \dots, \hat{P}_\sigma, \dots, \hat{P}_{N^2-1}$, as

$$V_P(\pi, \pi^*) = \sum_\sigma [\Phi_\sigma^i(\pi, P) \frac{\delta}{\delta\pi^i} + \Phi_\sigma^{i*}(\pi, P) \frac{\delta}{\delta\pi^{i*}}] \epsilon^\sigma. \quad (6.4)$$

Then the differential of some differentiable function $F(\pi, \pi^*)$ is

$$\delta_P F(\pi, \pi^*) = D_\sigma(\hat{P}) F(\pi, \pi^*) \epsilon^\sigma, \quad (6.5)$$

and, in particular, we have

$$\delta_P \pi^i = \Phi_\sigma^i(\pi, P) \epsilon^\sigma, \quad \delta_P \pi^{i*} = \Phi_\sigma^{i*}(\pi, P) \epsilon^\sigma. \quad (6.6)$$

For example, realizing rotations $\hat{s}_x, \hat{s}_y, \hat{s}_z$ from $AlgSU(2)$, one has

$$D_x(s) = -\frac{\hbar}{2} \left[[1 - \pi^2] \frac{\delta}{\delta\pi} - [1 - \pi^{*2}] \frac{\delta}{\delta\pi^*} \right],$$

$$\begin{aligned}
D_y(s) &= \frac{\hbar}{2} \left[[1 + \pi^2] \frac{\delta}{\delta\pi} + [1 + \pi^{*2}] \frac{\delta}{\delta\pi^*} \right], \\
D_z(s) &= \hbar \left[-\pi \frac{\delta}{\delta\pi} + \pi^* \frac{\delta}{\delta\pi^*} \right].
\end{aligned} \tag{6.7}$$

Then, we have well known commutation relations

$$[D_\mu(s), D_\nu(s)]_- = -i\hbar\epsilon_{\mu\nu\sigma}D_\sigma(s). \tag{6.8}$$

For a three-level system, the realization of a dynamical $SU(3)$ group symmetry is provided by an 8-dimensional local vector field [6], where $\hat{\lambda}_1, \dots, \hat{\lambda}_8$ are the Gell-Mann matrices, i.e.

$$\begin{aligned}
D_1(\lambda) &= i\frac{\hbar}{2} \left[[-1 + (\pi^1)^2] \frac{\delta}{\delta\pi^1} + \pi^1\pi^2 \frac{\delta}{\delta\pi^2} + [-1 + (\pi^{1*})^2] \frac{\delta}{\delta\pi^{1*}} + \pi^{1*}\pi^{2*} \frac{\delta}{\delta\pi^{2*}} \right], \\
D_2(\lambda) &= i\frac{\hbar}{2} \left[[1 + (\pi^1)^2] \frac{\delta}{\delta\pi^1} + \pi^1\pi^2 \frac{\delta}{\delta\pi^2} - [1 + (\pi^{1*})^2] \frac{\delta}{\delta\pi^{1*}} - \pi^{1*}\pi^{2*} \frac{\delta}{\delta\pi^{2*}} \right], \\
D_3(\lambda) &= -\frac{\hbar}{2} \left[\pi^2 \frac{\delta}{\delta\pi^2} + \pi^{2*} \frac{\delta}{\delta\pi^{2*}} \right], \\
D_4(\lambda) &= \frac{\hbar}{2} \left[[-1 + (\pi^2)^2] \frac{\delta}{\delta\pi^2} + \pi^1\pi^2 \frac{\delta}{\delta\pi^1} + [-1 + (\pi^{2*})^2] \frac{\delta}{\delta\pi^{2*}} + \pi^{1*}\pi^{2*} \frac{\delta}{\delta\pi^{1*}} \right], \\
D_5(\lambda) &= \frac{\hbar}{2} \left[[1 + (\pi^2)^2] \frac{\delta}{\delta\pi^2} + \pi^1\pi^2 \frac{\delta}{\delta\pi^1} - [1 + (\pi^{2*})^2] \frac{\delta}{\delta\pi^{2*}} - \pi^{1*}\pi^{2*} \frac{\delta}{\delta\pi^{1*}} \right], \\
D_6(\lambda) &= -\frac{\hbar}{2} \left[\pi^2 \frac{\delta}{\delta\pi^1} + \pi^1 \frac{\delta}{\delta\pi^2} - \pi^{2*} \frac{\delta}{\delta\pi^{1*}} - \pi^{1*} \frac{\delta}{\delta\pi^{2*}} \right], \\
D_7(\lambda) &= -\frac{\hbar}{2} \left[\pi^2 \frac{\delta}{\delta\pi^1} - \pi^1 \frac{\delta}{\delta\pi^2} - \pi^{2*} \frac{\delta}{\delta\pi^{1*}} + \pi^{1*} \frac{\delta}{\delta\pi^{2*}} \right], \\
D_8(\lambda) &= 3\hbar \left[\pi^2 \frac{\delta}{\delta\pi^2} - \pi^{2*} \frac{\delta}{\delta\pi^{2*}} \right].
\end{aligned} \tag{6.9}$$

In each of N charts of the local coordinates (3.1) these vector fields might be distinguished to two parts : Goldstone subspace B and Higgs subspace H with commutation relations of Z_2 -graded algebra $AlgSU(N)$: $[\hat{H}, \hat{H}]_- \subset H$, $[\hat{H}, \hat{B}]_- \subset B$, $[\hat{B}, \hat{B}]_- \subset H$ like ordinary (state-independent) elements of $AlgSU(N)$ [6-9].

In order to establish relationship between “internal” parameters of the droplet and “external” propagation of the scalar field near the light cone in the “reference spacetime”, we should “lift” a geodesic cyclic virtual transition in $CP(\infty)$ (as a model of a single particle) into the fiber bundle.

Namely, if we assume that *in accordance with the “superequivalence principle” an infinitesimal geodesic “shift” of dynamical variables could be compensated by an infinitesimal transformations of the basis in Hilbert space, then one can get some effective self-interaction potential as an addition to the mass term in original Klein-Gordon equation in the Lommel’s form (5.3)*. We will label hereafter vectors of the Hilbert space by Dirac’s notations $|\dots\rangle$ and tangent vectors to $CP(N-1)$ or $CP(\infty)$ by arrows over letters, $\vec{\xi}$, for example. Then one has a definition of the rate of a state vector changing $|v(x)\rangle = -(i/\hbar)\hat{P}|\Psi(x)\rangle$. Of course, any dynamical variable of the

scalar field, charge, for example, defines a rate of change of the state vector. For us it is interesting now to consider an “evolution” during the quantum transition along geodesic between vacuum state $|\Phi_0\rangle$ and the state vector (5.4). This “evolution” corresponds to a fast “proper time” τ [23] which is associated with frequency which should be close to the meson mass if one want to have a spatial propagation of the droplet close to the classical radius of electron.

The “descent” of the vector field $|v(x)\rangle$ onto the base manifold $CP(\infty)$ is a mapping by the formula

$$\begin{aligned} f_{*(\Psi^0, \dots, \Psi^m, \dots)}|v(x)\rangle &= \frac{d}{d\tau} \left(\frac{\Psi^1}{\Psi^0}, \dots, \frac{\Psi^i}{\Psi^0}, \dots \right) \Big|_0 \\ &= -(i/\hbar) [P_0^1 + (P_k^1 - P_k^0 \pi^1) \pi^k, \dots, P_0^i + (P_k^i - P_k^0 \pi^i) \pi^k, \dots] \\ &= \vec{\xi} \in T_\pi CP(\infty). \end{aligned} \quad (6.10)$$

If (and only if) one starts from the “vacuum” state in an arbitrary direction in $CP(\infty)$, i.e. from zeroth local coordinates $\pi^1 = \dots = \pi^i = \dots = 0$ one has

$$f_{*(1,0,\dots,0,\dots)}|v_0\rangle = -(i/\hbar) [P_0^1, \dots, P_0^i, \dots] \quad (6.11)$$

and, therefore, one can identify $P_0^i = f^i$ or $\hat{P} = \hat{B}$. In the general case this is not correct. In order to find concrete values of f^i for the “evolution” of the vacuum state toward our solution, we must use the formula $\pi^i(\tau) = R(f^i/g) \tan \Theta$. If state $\langle \Phi|$ is not so far from $\langle \Phi_0| = (1, 0, 0, \dots, 0)$ we can span them by an unique geodesic:

$$g^{-1}(1, 0, 0, \dots, 0) \hat{T}(\tau, g) = R^{-1}(\Phi^0, \Phi^1, \dots, \Phi^N). \quad (6.12)$$

Then one has $\cos \Theta = |\Phi^0|/R$, $|f^i| = g|\Phi^i|(R^2 - |\Phi^0|^2)^{-1/2}$ and $\arg f^i = \arg \Phi^i$ (up to the general phase). Thus we know f-elements from (5.6) for the transformation of the vacuum vector into the solution of the Lommel equation. The transformation of this solution into the vacuum vector is induced by elements of matrix of the general “polarization operator”

$$\hat{P} = \hat{G}^{-1}(\Phi) \hat{B}(\Phi) \hat{G}(\Phi) \quad (6.13)$$

Note, that complicated form of the matrix \hat{P} is the consequence of the fact that subgroup $H = U(1) \otimes U(N-1)$ is not the normal (invariant) subgroup of the group $SU(N)$. This operator determines a tangent vector field $\vec{\xi}$ (6.10). At a point $\pi + \delta\pi$ in $CP(\infty)$ the “shifted” field

$$\vec{\xi} + \delta\vec{\xi} = \vec{\xi} + \frac{\delta\vec{\xi}}{\delta l} \delta l \quad (6.14)$$

contains the derivative $\frac{\delta\vec{\xi}}{\delta l}$, which is not, in the general case, a tangent vector to $CP(\infty)$, but the *covariant derivative*

$$\frac{\Delta \xi^i}{\delta l} = \frac{\delta \xi^i}{\delta l} + \Gamma_{km}^i \xi^k \frac{\delta \pi^m}{\delta l}, \quad c.c. \quad (6.15)$$

is a tangent vector to $CP(\infty)$. Now we should “lift” the new tangent vector $\xi^i + \Delta\xi^i$ into the original Hilbert space \mathcal{H} , that is, one needs to realize two mappings: $f^{-1} : CP(\infty) \rightarrow \mathcal{H}$

$$\begin{aligned} & f^{-1}(\pi^1 + \Delta\pi^1, \dots, \pi^i + \Delta\pi^i, \dots) \\ &= [\Psi^0, \Psi^0(\pi^1 + \Delta\pi^1), \dots, \Psi^0(\pi^i + \Delta\pi^i), \dots] \\ &= [\Psi^0, \Psi^1 + \Psi^0\Delta\pi^1, \dots, \Psi^i + \Psi^0\Delta\pi^i, \dots] \end{aligned} \quad (6.16)$$

and then

$$\begin{aligned} f_{*\pi+\delta\pi}^{-1}(\vec{\xi} + \Delta\vec{\xi}) &= \frac{d}{d\tau}[\Psi^0, \Psi^0(\pi^1 + \Delta\pi^1), \dots, \Psi^0(\pi^i + \Delta\pi^i), \dots] \Big|_0 \\ &= [v^0, v^1 + \frac{d}{d\tau}(\Psi^0\Delta\pi^1) \Big|_0, \dots, v^i + \frac{d}{d\tau}(\Psi^0\Delta\pi^i) \Big|_0, \dots]. \end{aligned} \quad (6.17)$$

Herein a non-parallel (in the general case) local vector field, corresponding to some local dynamical variable like (6.2), arises along our geodesic. Our aim is to find the total field mass, charge etc. In order to do this we should use the parallel transport of the dynamical variables [27,6-9,11].

It may be shown in our original Hilbert space \mathcal{H} that the term $|dv\rangle$ arises as an additional rate of a change of some general state vector $|\Psi\rangle$

$$\begin{aligned} |dv\rangle &= -(i/\hbar)d\hat{P}|\Psi\rangle \\ &= [0, \frac{d}{d\tau}(\Psi^0\Delta\pi^1) \Big|_0 |1, x\rangle, \dots, \frac{d}{d\tau}(\Psi^0\Delta\pi^i) \Big|_0 |i, x\rangle, \dots], \end{aligned} \quad (6.18)$$

where $\Delta\pi^i = -\Gamma_{km}^i \xi^k d\pi^m \tau$. Then $\langle \Psi | d\hat{P} | \Psi \rangle$ may be treated as an “instantaneous” self-interacting potential of the scalar droplet associated with the infinitesimal gauge transformation of the local frame with coefficients (4.1). *That is self-preservation of the droplet (unperturbed geodesic sequence of virtual transitions) may be achieved by the radiation of the gauge (compensation) field due to the renormalization of dynamical variables and “rotation” of the ellipsoid of polarization.*

In order to find the additional terms to the Lagrangian density (5.1) induced by infinitesimal gauge transformations of the local frame in the tangent space to $CP(N-1)$ (6.16), one should take into account the fact that Fourier components in (5.5) do not depend on spacetime coordinates in the case of the “Lorentz-radial” symmetry. That is, only spacetime derivatives of the basis Hermitian functions

$$\begin{aligned} \nabla|n, y\rangle &= -\frac{2\vec{x}\exp(-y^2/2)}{r_0^2\sqrt{2^n n!}\sqrt{\pi}}(yH_n(y) - 2nH_{n-1}(y)), \\ \frac{\partial|n, y\rangle}{\partial t} &= \frac{2c^2 t \exp(-y^2/2)}{r_0^2\sqrt{2^n n!}\sqrt{\pi}}(yH_n(y) - 2nH_{n-1}(y)) \end{aligned} \quad (6.19)$$

arise in the formula for Lagrangian density which is induced by the “geodesic variation” of the initial Lagrangian (5.1). On the other hand only Fourier components (5.5)

are subjected to the variation by the geodesic flow. The state vector (6.14) inherits the geometric structure of the $CP(\infty)$ and perturbed Lagrangian as follows:

$$\begin{aligned}\mathcal{L}' = \mathcal{L}(\Psi + \Delta\Psi) &= (\Psi + \Delta\Psi)^{m*} \frac{\partial \langle m, y |}{\partial t} \frac{\partial |n, y \rangle}{\partial t} (\Psi + \Delta\Psi)^n \\ &\quad - (\Psi + \Delta\Psi)^{m*} \nabla \langle m, y | \nabla |n, y \rangle (\Psi + \Delta\Psi)^n \\ &\quad - \alpha^2 (\Psi + \Delta\Psi)^{m*} \langle m, y | n, y \rangle (\Psi + \Delta\Psi)^n,\end{aligned}\quad (6.20)$$

where $\Delta\Psi^i = -\Psi^0 \Gamma_{km}^i \xi^k d\pi^m \tau$.

It is useful to compare the new Lagrangian with the well known Lagrangians of both abelian

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial - ieA_\mu) \psi^* (\partial - ieA^\mu) \psi - \frac{1}{4} \lambda (|\psi|^2 - F^2)^2 \quad (6.21)$$

and non-abelian

$$\mathcal{L}_{\mathcal{NA}} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} (D_\mu \psi^{*a}) (D^\mu \psi^a) - \frac{1}{4} \lambda (\psi^{*a} \psi_a - F^2)^2 \quad (6.22)$$

Higgs models. Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g' \epsilon_{abc} A_\mu^b A_\nu^c$ are field tensors, $D_\mu \psi^a = \partial_\mu \psi^a + g' \epsilon_{abc} A_\mu^b \psi^c$ is the so-called “covariant derivative” and F is the “modulus of the vacuum”. There are some important differences between our Lagrangian (6.20) and Lagrangians (6.21),(6.22):

1. First of all we have a single fundamental self-interacting scalar field Φ and modes of this field correspond to the energy of one of the N topological vacuums (the choice of the vacuum is, as a matter of fact, the choice of the chart of the base manifold, the projective Hilbert space $CP(N-1)$).

2. Instead of three parameters (F, λ, g') of the models (6.21),(6.22), we have only one free parameter, the radius R of the sectional curvature of the projective Hilbert state space.

3. Terms which arise in the Lagrangian (6.20) under geodesic variation depend on *relative amplitudes* of the scalar field and they are connected with quantum transitions between different modes of this field. One can relate these terms to some gauge “surrounding field”.

4. Local “non-abelian” gauge transformations in the tangent bundle contain *true covariant derivatives relative to the Fubini-Study metric* in $CP(\infty)$ or, in some approximation, in $CP(N-1)$.

5. The new Lagrangian (6.20) looks like the Lagrangian of the classical field but it should be treated as a Lagrangian of a quantum field as it is obtained from the quantum projective state space.

6. The new Lagrangian gives the “Higgs mechanism” due to form of the connection (4.1) in $CP(N-1)$ in an absolutely natural way.

The equation of motion of the self-interacting field configuration (droplet) may be obtained from variation of the Lagrangian (6.20) relative to the variation of $|\Psi\rangle$.

One has a generalized Klein-Gordon equation

$$\begin{aligned} & \frac{\partial^2(\Psi + \Delta\Psi)^*}{\partial(x^0)^2} - \nabla^2(\Psi + \Delta\Psi)^* + \alpha^2(\Psi^* + \Delta\Psi^* + \Psi^* \frac{\delta\Delta\Psi}{\delta\Psi}) \\ & + \frac{\partial(\frac{\partial\Psi^*}{\partial x^0} \frac{\delta\frac{\partial\Delta\Psi}{\partial x^0}}{\delta\frac{\partial\Psi}{\partial x^0}})}{\partial x^0} - \frac{\partial(\frac{\partial\Psi^*}{\partial x^1} \frac{\delta\frac{\partial\Delta\Psi}{\partial x^1}}{\delta\frac{\partial\Psi}{\partial x^1}})}{\partial x^1} - \frac{\partial(\frac{\partial\Psi^*}{\partial x^2} \frac{\delta\frac{\partial\Delta\Psi}{\partial x^2}}{\delta\frac{\partial\Psi}{\partial x^2}})}{\partial x^2} - \frac{\partial(\frac{\partial\Psi^*}{\partial x^3} \frac{\delta\frac{\partial\Delta\Psi}{\partial x^3}}{\delta\frac{\partial\Psi}{\partial x^3}})}{\partial x^3} = 0. \end{aligned} \quad (6.23)$$

This equation is local in $CP(N-1)$ because this is connected with the local topological vacuum $\Psi^0 \neq 0$. An analogous equation may be written in every sheet of the atlas. One can represent $\Delta\Psi$ for enough small τ with following Fourier coefficients

$$\Delta\Psi^i = -\frac{g\Psi^0\tau^2}{\sqrt{1 + \frac{|\Psi^0|^2}{R^2}}} \Gamma_{km}^i \xi^k \Psi^m. \quad (6.24)$$

It is easily to see that if the radius R of the sectional curvature $1/R^2$ of the projective Hilbert space goes to infinity, one obtains the ordinary Klein-Gordon equation. In the general case the curvature of the projective state space influences the wave dispersion of a nonlinear solution of the equation (6.23). This may be treated as a base of the experimental testing of a quantum nonlinearity in the sense, which was mentioned above. This topic will be investigated in the near future.

If equation (6.23) possesses localizable solutions like solitons then one can treat such solutions as *primordial nonlocal elements of quantum theory* instead of “material points”. Furthermore, equation (6.23) effectively describes propagation of the self-interacting scalar field in a curved “dynamical spacetime”. That is the curvature of the projective state space may be related with the curvature of the Einstein’s “dynamical spacetime”, i.e. with gravity. This problem requires developments which will be discussed elsewhere.

7 Reality in quantum theory

Physicists can not deny the “reality” of some process as an independent essentiality “between measurements”. Everyone now agrees that a underlying structure of any process has quantum content. Since we can not explain local character of reaction in a measurement, we should accept the hypothesis of the “wave packet reduction (collapse)”. That is measurement plays a special role in the quantum case. It leads to the necessity of artificial separation of any natural quantum process on unitary motion-“evolution” that is governed, say, by the Schrödinger equation, and non-unitary motion-“measurement”. This difficulty seems to be unavoidable because in accordance with the commonly accepted point of view *only a macroscopic “device”*

creates the reality by projection of a quantum state onto a basis (functional frame). But the EPR paradox shows that reality of state of a remote quantum subsystem depends on the type of manipulation of a different subsystem even at an arbitrary long spacetime distance. So we have an incompatibility between foundations of the ordinary quantum mechanics and relativity.

We have proposed a new approach which we called “superrelativity” in order to avoid this crucial difficulty of quantum theory. It is an essentially non-linear scheme. We show that it may be successful in spite of the general criticism of non-linear approaches in the framework of the probabilistic interpretation [28].

Recently Aharonov, Anandan and Vaidman [29-31] tried to return to an early, but unfortunately, short-lived ontological interpretation of the wave function for a single quantum particle. These efforts seem may be very fruitful in a *nonlinear realization* of the main Schrödinger idea of a wave description of an “isolated” quantum system [6-11]. We should emphasize that our approach quite differs from Weinberg’s version of nonlinear quantum mechanics [16,17]. Namely, *we assume a nonlinear character of the theory may be observed only on subatomic distances but this nonlinearity should play a crucial role for the whole structure of the quantum theory. In particular, we should avoid a statistical hypothesis. Instead, we investigate the problem of description of a quantum nonlocal isolated system.*

It is useful to analyse our arguments for the introduction of a non-linear modification of quantum theory, comparing them with arguments against the description of a single quantum particle as they have been described in Ref. [29-31].

‘(i) We have never been seen the quantum state of a single particle in a laboratory. Indeed, while a wave is typically spread over a region of space, we never see a particle simultaneously in several distinct locations.’

This is absolutely correct if one means that state of the particle is a linear wave. But probably this is only a good linear approximation to the true non-linear structure of a quantum particle.

‘(ii) If we could see a quantum state, we could presumably distinguish it from any other quantum state, but the unitary time evolution of states in quantum mechanics implies that it is impossible to distinguish between two different non-orthogonal states. Different outcomes of a measurement distinguishing these two states correspond to orthogonal quantum states of the composite system (measuring device plus particle). But, the scalar product between the initial states was not zero and remains nonzero under unitary time evolution.’ *States of a particle in the non-linear version of quantum theory we study here are determined by the shape of the “ellipsoid of polarisation” as function of local coordinates. Different shapes of the “ellipsoid of polarization” correspond in the general case to non-orthogonal states vectors in ordinary Hilbert space. These are distinguishable in any point of $CP(N - 1)$. This space arises as a consequence of the fact that a true unitary evolution, which leaves the “vacuum” state invariant, is induced by transformations from the coset*

$SU(N)/S[U(1) \otimes U(N-1)] = CP(N-1)$. Therefore the Fubini-Study metric and connection in $CP(N-1)$ give a possibility to distinguish even two non-orthogonal (from the “point of view” of the metric in the original Hilbert space) state vectors.

‘(iii) If we associate physical reality with a spread-out wave then the instantaneous “collapse” of the wave to a point during a position measurement seems to conflict with relativity.’

In order to avoid this difficulty we have studied the non-linear Klein-Gordon equation which presumably has a soliton-like solution.

That is I try to build a dynamical model of a non-linear physical field in the spirit de Broglie-Schrödinger-Bohm. Therefore one can assume that just Fourier components in same global (in original Hilbert space) functional frame, associated with a measuring device, have a physical sense. In our case we used relative Fourier components π^i for a non-linear realization of the state space because in the linear case one has the old problem of the spreading wave packet which can not play the role of the carrier of the dynamical variables. In our case one can hope that it is possible to obtain some concentrated soliton-like solutions.

Secondly, if one accepts the idea that it is impossible to distinguish internal and external degrees of freedom of a quantum particle, then one should use a *local moving frame, related with own physical field of the particle* (in our case they are relative Fourier components π^i) instead of the global basis. We can say that the choice of both the field model (in our case it is classical scalar field) and the “vacuum” - chart in $CP(N-1)$ take the place of a “measuring device”. This requires “internal” local dynamical variables which can be measured, i.e., they must be comparable with ordinary (global) dynamical variables.

In both SR and GR we deal with the motion of a system of material points. In particular, in GR, one can think of a description in terms of the local tangent space (freely falling frame) at every point. From this point of view, the dynamical variable, constructed in each tangent space, depend on the state of the system (the spacetime neighbourhood).

If one takes into account this type of structure in the the framework of quantum theory, we see that its manifestation in GR is a spacial case of this more general (quantum) structure. This structure corresponds to a quantum theory on quite general fiber bundle where there is *dependence of quantum dynamical variables on the a quantum state*. From this point of view *SR and GR are merely particular cases of a general (quantum) structure*.

In the ordinary quantum scheme the process of propagation of the Schrödinger wave function exhibits misty properties. Without a measuring device (projective postulate) it is impossible to say something about the “existence” or “reality” of a carrier of the dynamical variables (particle). But in our case the evolution of quantum state of the droplet is reflected in $CP(N-1)$ and in any point of this base manifold we have absolutely defined *local in $CP(N-1)$ dynamical variables like (6.7)*. One can

build a local “internal moving frame” and relate states of the system to this frame. That is one does not need some *external device* for the reference frame. That is we can project state of the non-local “droplet” (as a set some “polarizations”) not onto an arbitrary chosen basis $\{|a\rangle\}_0^{N-1}$, associated with some measuring device, but onto its own local frame, associated with a field configuration. It is the well known the “method of a moving frame” of Cartan which has been used in the method of phenomenological Lagrangians.

The connection (4.1) in $CP(N-1)$ gives us a possibility to compare states in an *internal manner*. For example, one can to compare gradients of two scalar droplets at different times instead of a comparison of the gradient and direction of some external field. I do not like to say that one can measure some physical parameter of a quantum system without an “external” device. It is impossible. I would like to say that the both Fubini-Study metric in $CP(N-1)$ (3.3) and the connection (4.1) create a desirable completeness of quantum theory because the notion of “state” now has absolutely objective sense (i.e. even without any measurement). That is we can return (in this aspect) to the classical situation.

8 Discussion

The physical structure of our approach forbids any combination of local dynamical variables (tangent vectors fields) if they are connected with different coherent states (points in the base manifold $CP(N-1)$). Therefore it is impossible to separate a dynamical variable from the physical carrier (“droplet” in our example). In that connection I think experimental testing of the “EPR-correlations” like [32] should be subjected a critical analysis very carefully in the spirit of interesting work of Caroline H. Thompson [33]. I mean that one should pay attention to a nontrivial topological structure of the state space of real quantum particles.

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References

- [1] P.Dirac, Proc.Roy.Soc., A **114**, 243 (1927).
- [2] P.Dirac, Proc.Roy.Soc., A **117**, 610 (1928).
- [3] E.Schrödinger, Naturwissenschaften,**14**, 664 (1926).
- [4] E.Schrödinger, Ann. Physik, **79**, 361 (1926).
- [5] E.Schrödinger, Ann. Physik, **79**, 489 (1926).
- [6] P.Leifer, *Dynamics of the spin coherent state, Ph.D. Thesis, Institute for Low Temperature Physics and Engineering of the UkrSSR Academy of Science, Kharkov*, 1990.
- [7] P.Leifer, Nonlinear Modification of Quantum Mechanics, Preprint TAUP 2337-96, (submitted for publication).
- [8] L.Horwitz and P.Leifer, The Semiclassical Limit from the Geometry of the Projective State Space, Preprint TAUP 2277-95 (submitted for publication).
- [9] P.Leifer, Quantum physics as a superrelativity, (submitted for publication).
- [10] Y.Ne'eman,Phys.Lett.A **186**, 5 (1994).
- [11] P.Leifer, Superrelativity as an element of a final theory, (submitted for publication).
- [12] D.Bohm,*Causality and Chance in Modern Physics*, (London, 1957).
- [13] A.M. Perelomov, *Generalized Coherent States and Their Applications*, Springer-Verlag, 1986.
- [14] S.Kobayashi and K.Nomizu,*Foundations of Differential Geometry,vol.II*, (Interscience Publishers, New York-London-Sydney, 1969).
- [15] R.Cirelli, A. Mania', L. Pizzocchero, Int.Mod.Phys.,**6** No.12, 2133 (1991).
- [16] S. Weinberg, Ann. Phys. (N.Y.) **194**, 336 (1989).
- [17] S. Weinberg, Phys.Rev.Lett. **62**, 485 (1989).
- [18] A.Shapere and F.Wilczek, J.Fluid Mech. **198**, 557 (1989).

- [19] M.Born and E.Wolf,*Principles of Optics*,
(Pergamon Press, Oxford, London, New-York, 1964).
- [20] L.de-Broglie, Comptes Rend.,**180**, 498 (1925).
- [21] Janke-Emde-Lösch,*Tafeln Höherer Funktionen*, Stuttgart (1960).
- [22] Yu.P.Rubakov and B.Saha, Found.Phys. **25**, No.12, 1723 (1995).
- [23] L.P.Horwitz, Found.Phys. **22**, 491 (1992).
- [24] M.M.Lam and C.Dewdney, Found.Phys. **24**, No.1, 3 (1994).
- [25] M.-T.Jeakel and S.Reynaud, Phys.Rev.Lett.**76**, 2407 (1996).
- [26] K.R.W. Jones, Ann.Phys. **233**, 295 (1994).
- [27] Y. Ne'eman, Found.Phys. **16**, 361 (1986).
- [28] J.Polchinski, Phys.Rev.Lett. **66**, 397 (1991).
- [29] Y.Aharonov, J.Anandan, L.Vaidman, Phys.Rev.A **47**, 4616 (1993).
- [30] Y.Aharonov and L.Vaidman, Phys.Lett. A **178**, 38 (1993).
- [31] Y.Aharonov, J.Anandan, L.Vaidman, Found.Phys. **26**, 117 (1996).
- [32] A.Aspect, J.Dalibard and G.Roger, Phys.Rev.Lett., **49**, No.25, 1804 (1982).
- [33] C.H.Thompson, Found.Phys.Lett., **9**, No.4, 357 (1996).